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Stable de Sitter vacua in N=2, D=5 supergravity

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Abstract

We find $5D$ gauged supergravity theories exhibiting stable de Sitter vacua. These are the first examples of stable de Sitter vacua in higher-dimensional ($D > 4$) supergravity. Non-compact gaugings with tensor multiplets and R-symmetry gauging seem to be the essential ingredients in these models. They are however not sufficient to guarantee stable de Sitter vacua, as we show by investigating several other models. The qualitative behaviour of the potential also seems to depend crucially on the geometry of the scalar manifold.

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1 Introduction

Cosmological observations [1], suggesting that our current universe has a small positive cosmological constant, have lead to a vigorous search for de Sitter vacua in string theory (see e.g. [2]) and more modestly in supergravity (see e.g [3–8]). Up to this day, the only examples of stable de Sitter vacua in extended ($N \geq 2$) supergravity were found by Fré et al in [5] in the context of $N = 2$ $D = 4$ gauged supergravity. Some very specific elements of 4D supergravity were used to construct these examples, some of which have no clear string theory origin (like for instance the de Roo-Wagemans angles). The embedding of their models in $N = 4$ $D = 4$ gauged supergravity, and all semi-simple gaugings of $N = 4$ $D = 4$ supergravity coupled to six vector multiplets were discussed in [7, 8], but all the de Sitter vacua turned out to be unstable. In view of these problems, we thought it might be instructive to look for de Sitter vacua in higher-dimensional gauged supergravity theories, in order to find out what general ingredients are necessary to guarantee the existence of stable de Sitter vacua.

In this paper we will focus on $5D$ $N = 2$ gauged supergravity for several reasons. First of all, it is very similar to $4D$ $N = 2$ in certain respects. They both allow an arbitrary number of vector- and hypermultiplets, and there exist beautiful relations between their respective scalar manifolds. On the other hand, tensor multiplets seem to be somewhat easier to introduce in $5D$, and there are no duality symmetries in $5D$, which makes the $5D$ theory simpler [1].

Besides this, there are some very good other reasons to study $5D$ gauged supergravity. An important motivation comes from the holographic principle, of which AdS/CFT [11] is a particularly nice realization. The best understood example is the famous correspondence between Type IIB string theory on $AdS_5 \times S^5$ and $N = 4$ Super Yang Mills theory. A lot can be learned just by looking at the $5D$ $N = 8$ SO(6) gauged supergravity (which is assumed to be a consistent truncation of IIB supergravity on $AdS_5 \times S^5$). For instance, Anti-de Sitter critical points of the potential imply, under suitable conditions, non-trivial conformal fixed points for the Yang Mills theory, and supergravity kink solutions that interpolate between Anti-de Sitter vacua correspond to renormalization group flows in the dual YM theory (see [12] for a short review). There has been a lot of speculation that similarly, de Sitter (quantum) gravity might be dual to some (still unknown) Euclidean conformal field theory (see e.g [13]). However, the correspondence is a lot less clear than in the Anti-de Sitter case (for some recent reviews see [14]). Studying gauged supergravities (with probably non-compact gauge groups) that have stable de Sitter vacua might give us some more clues about a possible dS/CFT correspondence.

Finally, from a more phenomenological point of view, we should note that various authors have suggested that the universe may have undergone a phase where it was effectively 5-dimensional, see e.g. [15, 16] and references therein, giving another reason to understand the vacuum structure of $5D$ ($N = 2$) gauged supergravity theories.

Our paper is organized as follows. In section 2 we repeat some elements of 5D gauged

[1] For interesting recent progress on the coupling of scalar-tensor multiplets to $4D$ $N = 2$ supergravity, see [9]. The coupling of a vector-tensor multiplet to supergravity was done in [10].

supergravity coupled to tensor and vector multiplets. We then study the potential corresponding to R-symmetry gauging in more detail in section 3, where we prove that $U(1)_R$ gauging does not give rise to stable de Sitter vacua. In section 4 we present the first examples of stable de Sitter vacua in 5D gauged supergravity. Tensors charged under a non-compact group and R-symmetry gauging seem to be crucial. The following section contains more examples of $5D$ $N = 2$ gauged supergravities with charged tensors and R-symmetry gauging. Unfortunately, these do not lead to stable de Sitter vacua. In section 6 we show that if we replace R-symmetry gauging by a specific $U(1)$ gauging of the universal hypermultiplet, we can also get stable de Sitter vacua. Finally, in the last section, we summarize our results and mention a few interesting directions for future research.

2 $D = 5, \mathcal{N} = 2$ gauged supergravity coupled to tensor and vector multiplets

2.1 The ungauged theory

The theory we consider is obtained by gauging $D = 5, \mathcal{N} = 2$ supergravity coupled to vector- and tensor multiplets. These theories are completely determined by a constant symmetric tensor $C_{\tilde{I}\tilde{J}\tilde{K}}$. In particular, the manifold $\mathcal{M}$, parameterized by the $\tilde{n}$ scalars in the theory, can be viewed as a hypersurface

$$N(h) = C_{\tilde{I}\tilde{J}\tilde{K}} h^{\tilde{I}} h^{\tilde{J}} h^{\tilde{K}} = 1 \tag{2.1}$$

of an ambient space with $\tilde{n}+1$ coordinates $h^{\tilde{I}}$. The geometry of this surface is referred to as 'very special geometry'. For more details on very special geometry, see appendix A.

The 'very special real' manifolds were classified in [17] in the case that $\mathcal{M}$ is a homogeneous space [2]. The symmetric spaces are a subclass of these, and where already found in [18] and [19]. They can be divided into two subclasses, depending on whether they are associated with Jordan algebras or not:

1. When $\mathcal{M}$ is associated with a Jordan algebra, there are two subclasses:

    - The 'generic' or 'reducible' Jordan class:

$$\mathcal{M} = SO(1,1) \times \frac{SO(\tilde{n}-1,1)}{SO(\tilde{n}-1)}, \qquad \tilde{n} \geq 1. \tag{2.2}$$

[2] Homogeneous manifolds are manifolds for which its isometry group works transitively (on the manifold). The group G of linear transformations of the $h^{\tilde{I}}$ that leave $C_{\tilde{I}\tilde{J}\tilde{K}}$ invariant is a subgroup of the isometry group $Iso(\mathcal{M})$, but it is not the whole isometry group in general. For instance, the symmetric non-Jordan family (2.3) has isometries that are not in G. Strictly speaking, only those homogeneous spaces for which G works transitively are classified. To the best of our knowledge there is no proof that there are no other homogeneous (or even symmetric) very special real spaces (for which $Iso(\mathcal{M})$ works transitively, but G not).

- The 'irreducible' or 'magical' Jordan class:

$$
\begin{aligned}
\mathcal{M} &= SL(3,\mathbb{R})/SO(3), && (\tilde{n}=5)\\
\mathcal{M} &= SL(3,\mathbb{C})/SU(3), && (\tilde{n}=8)\\
\mathcal{M} &= SU^*(6)/USp(6), && (\tilde{n}=14)\\
\mathcal{M} &= E_{6(-26)}/F_4. && (\tilde{n}=26)
\end{aligned}
$$

2. There is one class which is not associated with a Jordan algebra, and which is therefore referred to as the 'symmetric non-Jordan family':

$$
\mathcal{M} = \frac{SO(1,\tilde{n})}{SO(\tilde{n})}, \qquad \tilde{n} > 1. \tag{2.3}
$$

The total global symmetry group of a matter coupled $\mathcal{N}=2$ supergravity theory factorizes into $SU(2)_R \times G$, where $SU(2)_R$ is the R-symmetry group of the theory and G is a group of linear transformations of the coordinates $h^{\tilde{I}}$ that leaves the tensor $C_{\tilde{I}\tilde{J}\tilde{K}}$ invariant. The symmetry group G gives rise to isometries of the scalar manifold. In the Jordan class, G even coincides with the full isometry group of $\mathcal{M}$.

2.2 Gauging the theory

The gauging of $\mathcal{N}=2$ supergravity coupled to n vector multiplets and m self-dual tensor multiplets was performed in [20–22]. The field content of the theory is

$$
\{e_\mu^m, \Psi_\mu^i, A_\mu^I, B_{\mu\nu}^M, \lambda^{i\tilde{a}}, \varphi^{\tilde{x}}\}\,, \tag{2.4}
$$

where

$$
\begin{aligned}
I &= 0,1,\ldots n\,,\\
M &= 1,2,\ldots 2m\,,\\
\tilde{I} &= 0,1,\ldots,n+2m\,,\\
\tilde{a} &= 1,\ldots,\tilde{n}\,,\\
\tilde{x} &= 1,\ldots,\tilde{n}\,,
\end{aligned}
$$

with $\tilde{n}=n+2m$. Note that we have combined the 'graviphoton' with the n vector fields of the n vector multiplets into a single $(n+1)$-plet of vector fields A_μ^I labeled by the index I. Also, the spinor and scalar fields of the vector and tensor multiplets are combined into $\tilde{n}$-tuples of spinor and scalar fields. The indices $\tilde{a},\tilde{b},\ldots$ and $\tilde{x},\tilde{y},\ldots$ are the flat and curved indices, respectively, of the $\tilde{n}$-dimensional target manifold $\mathcal{M}$ of the scalar fields. We also combine the vector and tensor indices I and M into one index $\tilde{I}$.

From the above fields, only the gravitini and the spin-1/2 fermions transform under the $SU(2)_R$ symmetry group. However, to gauge this group we need vectors that transform in the adjoint representation of the gauge group. This problem can be solved by identifying

the $SU(2)_R$ group with an $SU(2)$ subgroup of the symmetry group G of the C_{IJK}, and to gauge both $SU(2)$ groups simultaneously. If you just gauge a $U(1)_R$ subgroup, this problem does not occur since the adjoint of $U(1)$ is the trivial representation. An arbitrary linear combination of the vector fields can be used as $U(1)_R$ gauge field. However, if you also gauge a subgroup K of G, the $U(1)_R$ gauge field has to be a linear combination of the K-singlet vector fields only.

The simultaneous gauging of the $U(1)_R$ or $SU(2)_R$ R-symmetry group and a subgroup $K \subset G$ introduces a scalar potential of the form

$$e^{-1}\mathcal{L}_{pot} = -g^2 P, \tag{2.5}$$

where $P := P^{(T)} + P^{(R)}$. $P^{(R)}$ arises from the gauging of $U(1)_R$ or $SU(2)_R$, whereas $P^{(T)}$ is due to the tensor fields transforming under the gauge group K (see (2.6)).

The potential $P^{(T)}$ can be written as [23] [3]

$$P^{(T)} = \frac{3\sqrt{6}}{16} h^I \Lambda_I^{MN} h_M h_N, \tag{2.6}$$

with Λ_{IN}^{M} the transformation matrices of the tensor fields under the gauge group K and

$$\Lambda_I^{MN} \equiv \Lambda_{IP}^{M}\Omega^{PN} = \frac{2}{\sqrt{6}}\Omega^{MR}C_{IRP}\Omega^{PN}, \tag{2.7}$$

with $\overset{\circ}{a}{}^{\tilde{I}\tilde{J}}$ being the inverse of $\overset{\circ}{a}_{\tilde{I}\tilde{J}}$. Ω^{MN} is the inverse of Ω_{MN}, which is a (constant) invariant antisymmetric tensor of the gauge group K

$$\Omega_{MN} = -\Omega_{NM}, \qquad \Omega_{MN}\Omega^{NP} = \delta_M^P. \tag{2.8}$$

For the potential $P^{(R)}$ we have the following general expression

$$P^{(R)} = -4\vec{P}\cdot\vec{P} + 2\vec{P}^x\cdot\vec{P}_x\,, \tag{2.9}$$

where $\vec{P} = h^I\vec{P}_I$ and $\vec{P}_x = h_x^I\vec{P}_I$ are vectors in $SU(2)$-space (see [22]). When we gauge the full R-symmetry group $SU(2)_R$ we have

$$\vec{P}_I = \vec{e}_I V\,, \tag{2.10}$$

where V is an arbitrary constant, and $\vec{e}_I$ are constants that are nonzero only for I in the range of the $SU(2)$ factor and satisfy

$$\vec{e}_I \times \vec{e}_J = f_{IJ}{}^K\vec{e}_K\,, \tag{2.11}$$

with $f_{IJ}{}^K$ the $SU(2)$ structure constants. From now on, we will use indices $A, B, \ldots$ for the $SU(2)$ factor. We can then take $\vec{e}_A\cdot\vec{e}_B = \delta_{AB}$ such that (using (A.10))

$$P^{(R)} = -4V^2 C^{AB\tilde{I}} h_{\tilde{I}}\delta_{AB}, \tag{2.12}$$

[3] We assume $C_{MNP} = C_{IJM} = 0$. More general tensor couplings with $C_{IJM} \neq 0$ are possible, see [22], but we will not consider these here.

where we defined

$$C^{\tilde{I}\tilde{J}\tilde{K}} \equiv \overset{\circ}{a}^{\tilde{I}\tilde{I}'} \overset{\circ}{a}^{\tilde{J}\tilde{J}'} \overset{\circ}{a}^{\tilde{K}\tilde{K}'} C_{\tilde{I}'\tilde{J}'\tilde{K}'}. \tag{2.13}$$

In the case of $U(1)_R$ gauging we have

$$\vec{P}_I = V_I \vec{e}\,, \tag{2.14}$$

where $\vec{e}$ is an arbitrary vector in $SU(2)$ space and V_I are constants that define the linear combination of the vector fields A^I_μ that is used as the $U(1)_R$ gauge field

$$A_\mu[U(1)_R] = V_I A^I_\mu. \tag{2.15}$$

They have to be constrained by

$$V_I f^I_{JK} = 0, \tag{2.16}$$

with f^I_{JK} being the structure constants of K. Using the very special geometry identities of appendix A, the $U(1)_R$ potential can be written as

$$P^{(R)} = -4 C^{IJ\tilde{K}} V_I V_J h_{\tilde{K}}. \tag{2.17}$$

Finally, we remark that when $\mathcal{M}$ is associated with a Jordan algebra [18], one has (componentwise)

$$C^{\tilde{I}\tilde{J}\tilde{K}} = C_{\tilde{I}\tilde{J}\tilde{K}} = \text{const.} \tag{2.18}$$

3 Exploring the R-symmetry potential

3.1 U(1) R-symmetry gauging leads to tachyonic de Sitter vacua

Theorem

Without charged tensors or hypers, the potential gets only a contribution from R-symmetry gauging. Unlike in $4D$, non-Abelian vector multiplets do not contribute a term to the potential [24, 25]. For the $U(1)_R$ case the potential is given by (2.17).

In our conventions, a critical point φ_c of the potential with $P^{(R)}(\varphi_c) > 0$ corresponds to a de Sitter vacuum. We will demonstrate that if such a de Sitter vacuum exists, it will always be unstable. To prove this, we need to calculate the matrix of second derivatives of the potential at the critical point. A critical point, by definition, obeys the following relation

$$\frac{\partial P^{(R)}}{\partial \varphi^x}(\varphi_c) = -4 C^{IJK} V_I V_J h_{K,x}|_{\varphi_c} = 0\,. \tag{3.1}$$

For the mass matrix we find

$$g^{yz} \frac{\partial P^{(R)}}{\partial \varphi^x \partial \varphi^z}\Big|_{\varphi_c} = \left(\tfrac{2}{3} P^{(R)} \delta^y_x - 8\sqrt{\tfrac{2}{3}} V_I V_J h^{Iu} h^{Jv} T_{uvx;z} g^{zy} \right) \Big|_{\varphi_c}, \tag{3.2}$$

where we have used (A.9), (A.13) and (3.1). We will now show that $V_I h^I_y(\varphi_c)$ is an eigenvector of this matrix. Equation (A.12) leads to

$$T_{uvx;y} h^{Iu} h^{Jv} h^{Ky} V_I V_J V_K = \sqrt{\tfrac{3}{2}} [h^{Iu} h^J_u V_I V_J V_K h^K_x - 2 V_I V_J h^{Iu} h^{Jv} T_{uv}{}^w T_{wyx} V_K h^{Ky}] \,. \tag{3.3}$$

Since, using (A.11) and (3.1), we have

$$V_I V_J h^{Iu} h^{Jv} T_{uv}{}^w |_{\varphi_c} = V_I h^I V_J h^{Jw} |_{\varphi_c} \,, \tag{3.4}$$

we finally obtain

$$\begin{aligned} T_{uvx;y} h^{Iu} h^{Jv} h^{Ky} V_I V_J V_K |_{\varphi_c} &= \sqrt{\tfrac{3}{2}} [h^{Iu} h^J_u V_I V_J - 2 (V_I h^I)^2] V_K h^K_x |_{\varphi_c} \,, \\ &= \sqrt{\tfrac{3}{8}} P^{(R)} V_K h^K_x |_{\varphi_c} \,. \end{aligned} \tag{3.5}$$

We thus find that $V_I h^I_y(\varphi_c)$ is indeed an eigenvector of the mass matrix with eigenvalue $-10 P^{(R)}(\varphi_c)/3$. Looking at equations (2.9) and (2.14) we see that if a critical point with $P^{(R)} > 0$ exists, then also $V_I h^I_y(\varphi_c) \neq 0$. This proves that in case of a de Sitter extremum, the mass matrix has always at least one negative eigenvalue.

Example

To illustrate our proof, we will give an example of a de Sitter vacuum obtained by $U(1)_R$ gauging. The Jordan symmetric spaces only lead to Anti-de Sitter or Minkowski vacua (see [24]), but we will show here that there are also a lot of models with de Sitter vacua.

Equations (2.17), (3.1) and (A.7) lead to

$$C^{IJK} V_J V_K |_{\varphi_c} = -\tfrac{1}{4} P(\varphi_c) h^I(\varphi_c) \,. \tag{3.6}$$

This is a necessary and sufficient condition for φ_c to be a critical point. Furthermore, for φ_c in the domain where a_{IJ} is positive definite, one can always perform a linear transformation on the h^I such that $h^I(\varphi_c) = (1, 0, \ldots, 0)$ and $a_{IJ}(\varphi_c) = \delta_{IJ}$. After this transformation the polynomial $N(h)$ will take the following form

$$N(h) = (h^0)^3 - \tfrac{3}{2} h^0 h^i h^j \delta_{ij} + C_{ijk} h^i h^j h^k \,, \qquad i = 1, \ldots, \tilde{n}, \tag{3.7}$$

which is called the canonical parametrization of $N(h)$. Equation (3.6) then becomes

$$C_{0JK} V_J V_K = -\tfrac{1}{4} P(\varphi_c) \,, \tag{3.8}$$

$$C_{ijk} V_j V_k - V_0 V_i = 0 \,, \tag{3.9}$$

with summation over repeated indices. Equation (2.17) however leads to

$$P^{(R)}(\varphi_c) = -4 C_{0JK} V_J V_K = -4 V_0^2 + 2 \sum_i V_i^2 \,, \tag{3.10}$$

such that (3.8) is automatically fulfilled. So, given a theory (a tensor C_{ijk}) , we look for a vector V_I that solves equation (3.9) and for which

$$-4V_0^2 + 2\sum_i V_i^2 < 0\,. \tag{3.11}$$

As we know, this is not possible for general C_{ijk}. However, one can construct a lot of examples. Take for example $C_{ijk} = 0$. Equation (3.9) in this case leads to $V_i = 0$, corresponding to anti-de Sitter vacua or $V_0 = 0$, corresponding to de Sitter vacua. To study the mass matrix, we look at the particular example $n = 1$. We then have the following polynomial

$$N(h) = (h^0)^3 - \tfrac{3}{2}h^0(h^1)^2\,. \tag{3.12}$$

The constraint $N = 1$ can be solved by

$$h^0 \;=\; \frac{\varphi}{2} + \frac{\sqrt{\varphi+\varphi^4}}{2\varphi} \tag{3.13}$$

$$h^1 \;=\; \sqrt{\frac{2}{3}}\left(-\frac{3}{2}\varphi + \frac{\sqrt{\varphi+\varphi^4}}{2\varphi}\right)\,. \tag{3.14}$$

The metric on the scalar manifold is (using (A.3))

$$g_{xy} = \frac{3+12\varphi^3}{4(\varphi^2+\varphi^5)}\,. \tag{3.15}$$

We restrict to the region $\varphi > 0$, which contains the point $h^I(\varphi_c = 1/2) = (1, 0)$ and where the metric is positive definite.

Taking $V_0 = 0$, equation (3.9) is fulfilled for arbitrary values of V_1 and equation (3.10) becomes

$$P^{(R)}(\varphi_c) = 2V_1^2 > 0\,. \tag{3.16}$$

Using the metric (3.15) and with $V_0 = 0$ we get for the potential

$$P^{(R)}(\varphi) = \frac{-1 - 8\varphi^3 - 40\varphi^6 + 12\varphi\sqrt{\varphi+\varphi^4} + 24\varphi^4\sqrt{\varphi+\varphi^4}}{2(\varphi+4\varphi^4)}V_1^2\,, \tag{3.17}$$

which indeed fulfils (3.16). Furthermore,

$$P^{(R)}_{,x}(\varphi_c) = 0\,, \tag{3.18}$$

and

$$g^{yz}P^{(R)}_{,x,z}|_{\varphi_c} = -\frac{20}{3}V_1^2\,, \tag{3.19}$$

which is indeed $-10P^{(R)}(\varphi_c)/3$ as stated in the theorem above.

3.2 de Sitter vacua from $\mathbf{SU(2)_R}$ gauging

For the known symmetric spaces, $SU(2)_R$ gauging never gives any critical points (see [26]). We show here that there are however also a lot of models with unstable de Sitter vacua. Proving that there are no stable de Sitter vacua seems to be somewhat more difficult than in the $U(1)_R$ case and we hope to come back on this in a future publication.

We start from a polynomial in the canonical parametrization. In order to gauge $SU(2)_R$, the polynomial should have an $SU(2)_G$ symmetry [26]. Without charged tensors this further restricts the coefficients C_{ijk} by [16]

$$C_{ABC} = 0\,, \qquad C_{AB\alpha} = c_\alpha \delta_{AB} \qquad C_{A\alpha\beta} = 0\,, \tag{3.20}$$

with c_α some arbitrary constants. We have split the indices $i = 1, \ldots, n$ as $i = (A, \alpha)$ with $A, B, \ldots \in \{1, 2, 3\}$ corresponding to the SU(2) factor of the gauge group. The $C_{\alpha\beta\gamma}$ are still unconstrained.

Using expression (2.12) for the $SU(2)_R$ potential, the equation analogous to (3.6) is

$$C^{ABI}\delta_{AB}|_{\varphi_c} = -P(\varphi_c)h^I(\varphi_c)\,. \tag{3.21}$$

We now assume that $h^I(\varphi_c) = (1, 0, \ldots, 0)$. Equation (3.21) then leads to the conditions

$$P^{(R)}(\varphi_c) = \frac{3}{2} \tag{3.22}$$

$$c_\alpha = 0\,, \quad \forall\, \alpha\,, \tag{3.23}$$

and therefore $C_{ABi} = 0$, $\forall\, i$. The first condition is again automatically fulfilled and tells us that all these critical points are de Sitter.

We now investigate the stability of these de Sitter vacua. Calculating the second derivative of the potential, we get

$$P_{,x;y} = -C^{ABI}{}_{;y}\delta_{AB}h_{I,x} - C^{ABI}\delta_{AB}h_{I,x;y}\,. \tag{3.24}$$

Using $h^I(\varphi_c) = (1, 0, \ldots, 0)$, $a_{IJ}(\varphi_c) = \delta_{IJ}$ and (A.8) we get for the second term in the critical point

$$-C^{ABI}\delta_{AB}h_{I,x;y}|_{\varphi_c} = \frac{2}{3}P^{(R)}(\varphi_c)g_{xy}(\varphi_c) = g_{xy}(\varphi_c)\,. \tag{3.25}$$

For the first term we also use (A.13), (A.12), (A.9) and (3.20), which gives after some calculation

$$-C^{ABI}{}_{;y}\delta_{AB}h_{I,x}|_{\varphi_c} = -2g_{xy}(\varphi_c) - \tfrac{4}{3}h^A_x h^B_y \delta_{AB}\,, \tag{3.26}$$

and therefore

$$g^{zy}P_{,x;y}|_{\varphi_c} = -\delta^z_x - \tfrac{4}{3}h^A_x h^{Bz}\delta_{AB}\,. \tag{3.27}$$

This matrix has the following eigenvectors

$$\begin{cases} h^A_x(\varphi_c) & \text{with eigenvalue } -\tfrac{7}{3}\,, \\ h^\alpha_x(\varphi_c) & \text{with eigenvalue } -1\,. \end{cases} \tag{3.28}$$

The de Sitter vacua are therefore always maxima of the potential.

4 Stable de Sitter vacua in 5D N=2 gauged supergravity: an example

The previous section made clear that $U(1)_R$ gauging alone cannot give rise to stable de Sitter vacua. We show here that adding tensor multiplets can change this. The gauging we study in this section was already performed in [23] for $\tilde{n} = 3$. They found de Sitter extrema, but did not check that they are stable. We generalize for arbitrary $\tilde{n}$ and show that the obtained de Sitter vacua are all stable.

We consider $\mathcal{N} = 2$ supergravity coupled to $\tilde{n}$ Abelian vector multiplets and with scalar manifold $\mathcal{M} = SO(\tilde{n}-1,1) \times SO(1,1)/SO(\tilde{n}-1), \tilde{n} \geq 1$. The polynomial can then be written in the following form

$$N(h) = 3\frac{\sqrt{3}}{2}\, h^0[(h^1)^2 - (h^2)^2 - \ldots - (h^{\tilde{n}})^2]\,. \tag{4.1}$$

With $x = 1, \ldots, \tilde{n}$, introducing

$$\eta_{xy} = \eta^{xy} = \mathrm{Diag}(1, -1, \ldots, -1)\,, \tag{4.2}$$

we write the $C_{\tilde{I}\tilde{J}\tilde{K}}$ symbols as

$$C_{0\underline{xy}} = \frac{\sqrt{3}}{2}\eta_{xy}\,. \tag{4.3}$$

(we underline x-type indices that are in fact of type $\tilde{I}$, but take values in the x-range due to our choice). The constraint $N = 1$ can be solved by

$$h^0 = \frac{1}{\sqrt{3}\|\varphi\|^2}\,, \qquad h^{\underline{x}} = \sqrt{\frac{2}{3}}\,\varphi^x\,,$$

with

$$\|\varphi\|^2 = \varphi^x\eta_{xy}\varphi^y\,. \tag{4.4}$$

The hypersurface $N = 1$ decomposes into three disconnected components:
(i) $\|\varphi\|^2 > 0$ and $\varphi^1 > 0$
(ii) $\|\varphi\|^2 < 0$
(iii) $\|\varphi\|^2 > 0$ and $\varphi^1 < 0$.
In the following, we will consider the "positive timelike" region (i) only, since in region (ii), $g_{\tilde{x}\tilde{y}}$ and $\overset{\circ}{a}_{\tilde{I}\tilde{J}}$ are not positive definite , and region (iii) is isomorphic to region (i).

We now proceed by gauging the above theory. The isometry group of the scalar manifold is $SO(\tilde{n}-1,1) \times SO(1,1)$. We gauge the noncompact subgroup $SO(1,1) \subset SO(\tilde{n}-1,1)$ together with $U(1)_R \subset SU(2)_R$. The $SO(1,1)$ subgroup rotates h^1 and h^2 into each other and therefore acts nontrivially on the vector fields A^1_μ and A^2_μ. In order for the resulting theory to be supersymmetric, these vectors have to be dualized to antisymmetric tensor fields. We can thus decompose the index $\tilde{I}$ in the following way

$$\tilde{I} = (I, M)\,, \tag{4.5}$$

with $I, J, K, \ldots = 0, 3, 4, \ldots, \tilde{n}$ and $M, N, P, \ldots = 1, 2$.

Furthermore, we need a vector transforming in the adjoint of $SO(1,1)$ (which means it should be inert) to act as its gauge field. Looking at $\Lambda_{IN}^M \sim C_{IRN}\Omega^{MR}$, we see that only A_μ^0 couples to the tensor fields (only $C_{0RP} \neq 0$) and thus acts as the gauge field of $SO(1,1)$. The remaining vectors are called 'spectator fields' with respect to the $SO(1,1)$ gauging.

Finally, for the $U(1)_R$ gauge field we take a linear combination $A_\mu[U(1)_R] = V_I A_\mu^I$ of the vectors. We now have the ingredients to calculate the potentials (2.6) and (2.17) (taking $\Omega^{12} = -\Omega^{21} = -1$):

$$\begin{aligned} \Lambda_{0N}^M &= \frac{2}{\sqrt{6}}\Omega^{MR}C_{0RN} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, && (4.6)\\ P^{(T)} &= \frac{3\sqrt{6}}{16}h^I\Lambda_I^{MN}h_M h_N = \frac{1}{8}\frac{(\varphi^1)^2-(\varphi^2)^2}{\|\varphi\|^6}, && (4.7)\\ P^{(R)} &= -4\sqrt{2}V_0V_i\varphi^i\|\varphi\|^{-2} + 2|V|^2\|\varphi\|^2, \qquad |V|^2 \equiv V_iV_i, && (4.8) \end{aligned}$$

where we defined a new index i as $I = (0, i)$. Then

$$P = P^{(T)} + P^{(R)} = \frac{1}{8}\frac{(\varphi^1)^2-(\varphi^2)^2}{\|\varphi\|^6} - 4\sqrt{2}V_0\varphi^iV_i\|\varphi\|^{-2} + 2\|\varphi\|^2|V|^2. \tag{4.9}$$

Demanding $P_{,\tilde{x}} = 0$ gives the following conditions on the critical points

$$\begin{aligned} \frac{\varphi^i}{\|\varphi\|^4} &= 16\sqrt{2}V_0V_i, && (4.10)\\ \frac{1}{\|\varphi\|^6} &= -\frac{1}{2}\left(16\sqrt{2}V_0|V|\right)^2 + 8|V|^2, && (4.11) \end{aligned}$$

with the constraints

$$\begin{aligned} &|V|^2 \neq 0, \\ &32V_0^2 < 1. && (4.12) \end{aligned}$$

From (4.11) we see that $\|\varphi\|^2$ is completely determined by the V_I. From (4.10) we see that also the φ^i are completely fixed by the V_I. This means that only φ^1 and φ^2 can still vary, as long as the combination $(\varphi^1)^2 - (\varphi^2)^2$ remains fixed. We thus have a one parameter family of critical points, which is due to the unbroken $SO(1,1)$. There is also an unbroken $SO(\tilde{n}-3)$, but the vacuum is at the symmetric point.

The value of the potential in the critical points becomes

$$P(\varphi_c) = 3\|\varphi\|^2|V|^2\left(1 - 32V_0^2\right), \tag{4.13}$$

which is clearly positive because of (4.12) and therefore corresponds to de Sitter vacua.

We now show that all these vacua are stable. We use the index $m = 1, 2$ for the scalars related to $M = 1, 2$. Calculating the second derivatives of the potential in the critical points

gives the following Hessian

$$
\begin{aligned}
P_{,m,n}(\varphi_c) &= (\eta\varphi)_m(\eta\varphi)_n \left[3\|\varphi\|^{-8} + 4\varphi^k\varphi^k\|\varphi\|^{-10}\right] , \\
P_{,m,i}(\varphi_c) &= -4(\eta\varphi)_m\varphi^i \left[\|\varphi\|^{-8} + \varphi^k\varphi^k\|\varphi\|^{-10}\right] , \\
P_{,i,j}(\varphi_c) &= \varphi^i\varphi^j \left[5\|\varphi\|^{-8} + 4\varphi^k\varphi^k\|\varphi\|^{-10}\right] + \tfrac{1}{4}\delta_{ij}\|\varphi\|^{-6} ,
\end{aligned}
\tag{4.14}
$$

where $(\eta\varphi)_x \equiv \eta_{xy}\varphi^y$.

The $SO(1,1)$ invariance implies a zero eigenvector $\varphi^n(\sigma_1)_n{}^m$. Using this $SO(1,1)$ and the $SO(\tilde{n}-2)$ of the φ^i, we may further use for any critical point $\varphi_c = (\varphi^1, 0, \varphi^3, 0, \ldots, 0)$ with $|\varphi^3| < |\varphi^1|$. Then the zero mode is φ^2, and the sector $\varphi^4, \ldots, \varphi^{\tilde{n}}$ decouples as a unit matrix times $\frac{1}{4}\|\varphi\|^{-6}$. The relevant part of the hessian therefore is

$$
\partial\partial P = |||\varphi|||^{-10} \begin{pmatrix} (\varphi^1)^2\,[3(\varphi^1)^2 + (\varphi^3)^2] & -4(\varphi^1)^3\varphi^3 \\ -4(\varphi^1)^3\varphi^3 & \frac{1}{4}\,[(\varphi^1)^4 + 18(\varphi^1)^2(\varphi^3)^2 - 3(\varphi^3)^4] \end{pmatrix} , \tag{4.15}
$$

where we defined $\partial\partial P \equiv \partial_{\tilde{x}}\partial_{\tilde{y}}P(\varphi_c)|_{\tilde{x},\tilde{y}=1,3}$. The determinant and trace are

$$
\det\partial\partial P = \tfrac{3}{4}(\varphi^1)^2\|\varphi\|^{-14} , \qquad \mathrm{Tr}\,\partial\partial P = \tfrac{1}{4}\|\varphi\|^{-10}\left[13(\varphi^1)^4 + 22(\varphi^1)^2(\varphi^3)^2 - 3(\varphi^3)^4\right] , \tag{4.16}
$$

which shows that the eigenvalues are positive.

Comments

- **BEH effect.** Like in [5,6], the massless scalar is a Goldstone boson. It will get 'eaten' by the SO(1,1) gauge field, making the gauge field massive. There will thus be only positive mass scalars left in the effective theory.

- **Quantized scalar masses.** The masses of the scalars are given by the eigenvalues of $g^{xy}P_{x,y}(\varphi_c)/P(\varphi_c)$. We already showed that these will be positive. Using Mathematica, the scalar mass spectrum turns out to be

$$
(0, \frac{8}{3}\frac{(\varphi_1)^2 - (\varphi_2)^2}{\|\varphi\|^2}, \frac{2}{3}, \frac{2}{3}, \ldots, \frac{2}{3}) . \tag{4.17}
$$

  In [4] it was observed that all known examples of de Sitter extrema in extended supergravities have scalar masses that are quantized in units of the cosmological constant. This is also true in our model for all scalars, but one. One of the scalar masses depends on the parameters V_I that determine the $U(1)_R$ gauge field. Also in [8] examples of (unstable) de Sitter extrema were found that had parameter-dependent scalar masses.

- $SU(2)_R$ **gauging.** Instead of gauging $U(1)_R$ we could also have gauged the full $SU(2)_R$ R-symmetry as long as there are a sufficient number of gauge fields available ($\tilde{n} \geq 5$). Without loss of generality, we can choose A^3_μ, A^4_μ, A^5_μ as the $SU(2)$ gauge fields. We then find

$$
P^{(R)} = \frac{3}{2}\|\varphi\|^2 . \tag{4.18}
$$

Looking at equation (4.8), we observe that we get the same potential if we do a $U(1)_R$ gauging with $V_0 = 0$ and $|V|^2 = 3/4$. $SU(2)_R$ gauging with tensors charged under $SO(1,1)$ therefore will also lead to stable de Sitter vacua. The scalar masses are in this case given by $(0, 8/3, 2/3, 2/3, ..., 2/3)$.

5 $U(1)_R$ gauging and charged tensors: more examples

To try to find out what ingredients are really necessary to obtain stable de Sitter vacua, we will now look at a few other examples with $U(1)_R$ and charged tensors. A natural idea is to take the scalar manifold $\mathcal{M}$ to be one of the other known symmetric very special real manifolds.

5.1 The magical Jordan family

5.1.1 $\mathcal{M} = SL(3,\mathbb{R})/SO(3)$

$\mathcal{M}$ is described as the hypersurface $N(h) = 1$ of the cubic polynomial [17, 26]

$$N(h) = \frac{3}{2}\sqrt{3}h^3\eta_{\alpha\beta}h^\alpha h^\beta + \frac{3\sqrt{3}}{2\sqrt{2}}\gamma_{\alpha MN}h^\alpha h^M h^N\,, \tag{5.1}$$

where

$$\begin{aligned}
\alpha, \beta &= 0, 1, 2\,, \qquad\quad M, N = 4, 5\,, \\
\eta_{\alpha\beta} &= diag(+,-,-)\,, \quad \gamma_0 = -\mathbb{1}_2 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\,, \\
\gamma_1 &= \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\,, \quad \gamma_2 = \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\,.
\end{aligned} \tag{5.2}$$

The vector field metric $a_{\tilde{I}\tilde{J}}$ becomes degenerate when $\eta_{\alpha\beta}h^\alpha h^\beta = 0$, so we can restrict ourselves to the region $\eta_{\alpha\beta}h^\alpha h^\beta \neq 0$. To solve the constraint $N(h) = 1$, we take the parametrization used in [26],

$$h^\alpha = \sqrt{\frac{2}{3}}x^\alpha\,,\; h^M = \sqrt{\frac{2}{3}}b^M\,,\; h^3 = \frac{1 - b^T\bar{x}b}{\sqrt{3}||x||^2}\,, \tag{5.3}$$

where $b^T\bar{x}b \equiv b^M\bar{x}_{MN}b^N$ with $\bar{x}_{MN} = x^\alpha\gamma_{\alpha MN}$ and $||x||^2 \equiv \eta_{\alpha\beta}x^\alpha x^\beta$. The metrics $a_{\tilde{I}\tilde{J}}$ and g_{xy} are only positive definite in the region $||x||^2 > 0$ and $x_0 > 0$. Since this is the physically relevant region, we will restrict ourselves to this domain from now on.

In this model we can gauge the $SO(2,1)$ symmetry between h^α with the vector fields A^α_μ, while dualizing the non-trivially charged vector fields A^M_μ to tensor fields. We then get a potential

$$P^{(T)} = \frac{1}{8}b^T\bar{x}\Omega\bar{x}\Omega\bar{x}b\,, \quad \Omega = i\sigma_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\,. \tag{5.4}$$

Gauging the full R-symmetry is not possible in this model, but we can gauge a $U(1)_R$ symmetry. We have $A_\mu[U(1)_R] = V_I A^I_\mu$, with $V_I f^I_{JK} = 0$. From this it follows that $A_\mu[U(1)_R] = V_3 A^3_\mu$, so A^3_μ is the $U(1)_R$ gauge field. Since $P^{(R)} = -4C^{IJ\tilde{K}} V_I V_J h_{\tilde{K}}$ and $C^{33\tilde{K}} = C_{33\tilde{K}} = 0$ we find $P^{(R)} = 0$. The total potential P is thus given by $P^{(T)}$ alone. The critical points of P are given by $b^M = 0$, leading to Minkowski vacua. They are supersymmetric when the $U(1)_R$ gauging is turned off ($V_3 = 0$). There are no de Sitter vacua in this model.

5.1.2 $\mathcal{M} = SL(3,\mathbb{C})/SU(3)$

$\mathcal{M}$ is described as the hypersurface $N(h) = 1$ of the cubic polynomial [17, 26]

$$N(h) = \frac{3}{2}\sqrt{3} h^4 \eta_{\alpha\beta} h^\alpha h^\beta + \frac{3\sqrt{3}}{2\sqrt{2}} \gamma_{\alpha MN} h^\alpha h^M h^N \,, \tag{5.5}$$

where

$$\begin{aligned} \alpha, \beta &= 0,1,2,3\,, \quad M, N = 5,6,7,8\,, \\ \eta_{\alpha\beta} &= diag(+,-,-,-)\,, \quad \gamma_0 = -\mathbb{1}_4\,, \\ \gamma_1 &= \mathbb{1}_2 \otimes \sigma_1\,, \quad \gamma_2 = \sigma_2 \otimes \sigma_2\,, \quad \gamma_3 = \mathbb{1}_2 \otimes \sigma_3\,. \end{aligned} \tag{5.6}$$

We take the same parametrization as in the previous model,

$$h^\alpha = \sqrt{\frac{2}{3}} x^\alpha\,,\ h^M = \sqrt{\frac{2}{3}} b^M\,,\ h^4 = \frac{1 - b^T \bar{x} b}{\sqrt{3}||x||^2}\,. \tag{5.7}$$

The metrics are again only positive definite in the region $||x||^2 > 0$ and $x_0 > 0$.

The model above has an $SO(3,1) \times U(1)$ symmetry, which acts on the fields $h^{\tilde{I}}$ (and similarly on the vector fields $A^{\tilde{I}}_\mu$) as [27]

$$\begin{aligned} \delta h^\alpha &= B^\alpha{}_\beta h^\beta\,, \\ \delta h^M &= \tfrac{1}{4} B^{\alpha\beta} (\gamma_{\alpha\beta})^M{}_N h^N + S^M{}_N h^N \epsilon\,, \end{aligned} \tag{5.8}$$

where

$$\begin{aligned} &S \equiv \gamma_1\gamma_2\gamma_3 = i\sigma_2 \otimes \mathbb{1}_2\,, \qquad S^2 = -\mathbb{1}_4\,, \\ &\gamma_{ab} = \gamma_{[a}\gamma_{b]} = -S\varepsilon_{abc}\gamma^c\,, \quad \gamma_{0a} = -\gamma_{a0} = \gamma_a\,, \quad a = 1,2,3\,. \end{aligned} \tag{5.9}$$

Indices on the matrices $B_{\alpha\beta}$ are raised and lowered with $\eta_{\alpha\beta}$ and these transformations satisfy $B_{\alpha\beta} = -B_{\beta\alpha}$. This implies that they describe SO$(1,3)$. The motivation for the definition of $\gamma_{0a} = -\gamma_{a0}$ is based on a larger Clifford algebra, see [27, (5.16)]. The γ-matrices are symmetric, while S is antisymmetric. ϵ is the parameter for the U(1) symmetry.

To gauge a symmetry, we have to assign the isometry transformations to vector multiplets, i.e. to connect the parameters of gauge transformations Λ^I to parameters of the isometry

group, such that the transformations on the vector part form the adjoint representation and on the tensor multiplets there exists an antisymmetric matrix Ω_{MN} such that (see (2.7))

$$\Omega_{MP}\Lambda_{IN}^{P} = \frac{2}{\sqrt{6}}C_{IMN}\,. \tag{5.10}$$

- $U(1)_R$ **gauging and tensors charged under** $U(1)\times SU(2)$.

  We now gauge the SO(3) part of (5.8) with $B^{0a}=0$ and take A_μ^a as the adjoint vectors. We gauge the U(1) by the vector field A_μ^0. To gauge this symmetry, we also have to dualize the vector fields A_μ^M to tensor fields. We have

$$B_{ab} = \alpha\varepsilon_{abc}\Lambda^c\,, \qquad \epsilon = \beta\Lambda^0\,, \qquad \varepsilon_{123} = 1\,, \tag{5.11}$$

  where we allowed for arbitrary coefficient α and β to be determined below. This leads thus to the transformation matrices

$$\Lambda_{0N}^{M} = \beta S^M{}_N\,, \qquad \Lambda_{aN}^{M} = -\tfrac{1}{2}\alpha(S\gamma_a)^M{}_N\,. \tag{5.12}$$

  Checking (5.10) gives

$$\Omega = \frac{1}{\alpha}S\,, \qquad \beta = \frac{\alpha}{2}\,. \tag{5.13}$$

  For simplicity, and without losing generality, we can choose $\alpha = 1$, $\beta = 1/2$.

  In [26] the vector fields A_μ^a where used to also gauge the full $SU(2)_R$ symmetry and it was found that the total potential has no critical points. Instead, we will use the vector field A_μ^4 together with A_μ^0 to gauge the $U(1)_R$ symmetry. The potential $P = P^{(T)} + P^{(R)}$ becomes

$$\begin{aligned} P^{(R)} &= -2\sqrt{3}V_0\left(\frac{V_0}{\sqrt{3}}||x||^2 + \frac{4}{\sqrt{3}}V_4\left(\frac{1-b^T\bar{x}b}{\sqrt{2}||x||^2}\right)x^0 - \frac{2}{\sqrt{6}}V_4 b^T b\right)\,, && (5.14)\\ P^{(T)} &= \frac{1}{8}b^T\bar{x}\Omega\bar{x}\Omega\bar{x}b = -\frac{1}{8}b^T\bar{x}^3 b \\ &= \frac{1}{8}||x||^2 b^T\bar{x}b - \frac{1}{2}x_0^2 b^T\tilde{x}b + \frac{1}{4}x_0||\tilde{x}||^2 b^T b + \frac{1}{4}x_0^3 b^T b\,, && (5.15) \end{aligned}$$

  where $b^Tb = b^M b^N\delta_{MN}$, $||\tilde{x}||^2 = x^a x^b\delta_{ab}$, $||x||^2 = \eta_{\alpha\beta}x^\alpha x^\beta$, $\bar{x}_{MN} = x^\alpha\gamma_{\alpha MN}$ and $\tilde{x}_{MN} = x^a\gamma_{aMN}$. The last line of (5.15) makes the $U(1)\times SU(2)$ symmetry of the potential manifest, since $b^T\bar{x}b$, $b^T\tilde{x}b$, $||\tilde{x}||^2$ and $||x||^2$ are easily seen to be invariant under the transformations (5.8) (with $B_{a0}=0$). Using this symmetry we can restrict the search for extrema to points where e.g. $x_2 = x_3 = b_8 = 0$. We analyzed the potential with Mathematica and found no de Sitter vacua (in the region where the metrics are positive-definite).

  When $b^M = 0$, finding critical points of P reduces to finding critical points of $P^{(R)}$. It was shown in [24] that $P^{(R)}$ has an Anti-de Sitter maximum if and only if $V^{\sharp\tilde{I}} \equiv \sqrt{\frac{2}{3}}C^{\tilde{I}\tilde{J}\tilde{K}}V_{\tilde{J}}V_{\tilde{K}}$ lies in the domain of positivity of the Jordan algebra, and a Minkowski

critical point if and only if $V^{\sharp\tilde{I}} = 0$ ($P^{(R)}$ identically zero). The total potential P also has these critical points, but the extra potential from the tensors can change the nature of these critical points (e.g from a maximum to a saddle point). With Mathematica we also found Anti-de Sitter vacua of P with $b^M \neq 0$. Since our primary interest was finding de Sitter vacua, we did not check the nature of these critical points.

- **$U(1)_R$ gauging and tensors charged under $U(1)\times SO(2,1)$**
  Instead of the compact symmetry above, we can also gauge $U(1)\times SO(2,1)$ by again dualizing the vector fields A_μ^M to tensor fields and choosing the $SO(2,1)$ gauge fields to be A_μ^0, A_μ^1, A_μ^3, while letting A_μ^2 correspond to the $U(1)$ gauge field. Similarly as in the previous example, this leads to $\Omega = \gamma_2 S$. We can again gauge the $U(1)_R$ symmetry, this time with a linear combination of A_μ^2 and A_μ^4. This leads to the following potential,

$$
\begin{aligned}
P^{(R)} &= 2\sqrt{3}V_2\left(\frac{V_2}{\sqrt{3}}||x||^2 - \frac{4}{\sqrt{3}}V_4\left(\frac{1-b^T\bar{x}b}{\sqrt{2}||x||^2}\right)x^2 + \frac{2}{\sqrt{6}}V_4 b^T\gamma_2 b\right)\,,\\
P^{(T)} &= \frac{1}{8}b^T\bar{x}\Omega\bar{x}\Omega\bar{x}b\\
&= -\frac{1}{8}||x||^2 b^T\bar{x}b - \frac{1}{2}x_2^2 b^T\tilde{x}b - \frac{1}{4}x_2||\tilde{x}||^2 b^T b - \frac{1}{4}x_2^3 b^T b\,,
\end{aligned}
\tag{5.16}
$$

  where now $||\tilde{x}||^2 = (x^0)^2 - (x^1)^2 - (x^3)^2$ and $\tilde{x}_{MN} = x^0\gamma_{0MN} + x^1\gamma_{1MN} + x^3\gamma_{3MN}$. Analyzing the potential as in the previous case, we again found no de Sitter vacua. The potential has a critical point only when $V_2 = 0$. $P^{(R)}$ is then identically zero, and we have a family of Minkowski vacua at $b^M = 0$.

5.1.3 The other magical Jordan symmetric spaces

The spaces $\mathcal{M} = SU^*(6)/USp(6)$ and $\mathcal{M} = E_{6(-26)}/F_4$ are 14 and 26 dimensional respectively, and allow for more possibilities to get charged tensor multiplets, making the potential more difficult to analyze. Because all the magical Jordan spaces have a similar structure, one might expect similar qualitative features as in the previous models, but this has to be checked in detail to be sure.

5.2 The non-Jordan symmetric spaces

We now consider theories with $\mathcal{M} = \frac{SO(1,\tilde{n})}{SO(\tilde{n})}, \tilde{n} > 1$. We can then take the following polynomial

$$
N(h) = \frac{3}{2}\sqrt{\frac{3}{2}}\left(\sqrt{2}h^0(h^1)^2 - h^1\left[(h^2)^2 + \ldots + (h^{\tilde{n}})^2\right]\right)\,. \tag{5.17}
$$

This means for the non-vanishing components of the tensor $C_{\tilde{I}\tilde{J}\tilde{K}}$

$$
C_{011} = \frac{\sqrt{3}}{2}\,,\quad C_{1xy} = -\frac{\sqrt{6}}{4}\,\delta_{xy}\,,\; x,y = 2,\ldots,\tilde{n}\,. \tag{5.18}
$$

The constraint $N = 1$ can be solved by

$$h^0 = \sqrt{\frac{2}{3}}\left(\frac{1}{\sqrt{2}(\varphi^1)^2} + \frac{1}{\sqrt{2}}\varphi^1\left[(\varphi^2)^2 + \ldots + (\varphi^{\tilde{n}})^2\right]\right), \tag{5.19}$$

$$h^1 = \sqrt{\frac{2}{3}}\varphi^1, \qquad h^x = \sqrt{\frac{2}{3}}\varphi^1\varphi^x. \tag{5.20}$$

The Lagrangian of the theory is not invariant under the full isometry group $SO(1,\tilde{n})$. Only the subgroup $G = [SO(\tilde{n}-1) \otimes SO(1,1)] \ltimes T_{\tilde{n}-1}$, with $T_{\tilde{n}-1}$ the group of translations in an $(\tilde{n}-1)$ dimensional Euclidean space, leaves the tensor $C_{\tilde{I}\tilde{J}\tilde{K}}$ invariant and can thus be gauged [28]. In order to gauge a subgroup $K \subset G$ we need $Dim(K)$ vectors transforming in the adjoint of K. Furthermore, we want an additional number of vectors transforming non-trivially under K. After dualization to tensor multiplets these give the required contribution $P^{(T)}$ to the potential.

The subgroup $SO(1,1)$ can not be gauged since all vectors transform non-trivially under this group and we need an invariant vector to gauge SO(1,1).

The group $SO(\tilde{n}-1)$ rotates $h^2, \ldots, h^{\tilde{n}}$ (and thus also the vectors $A^2_\mu, \ldots, A^{\tilde{n}}_\mu$) into each other. This means that only its subgroup $SO(2)$ can be gauged in order to have both vectors that transform in the adjoint and vectors that transform non-trivially but not in the adjoint of the gauge group. We will therefore gauge this $SO(2)$, possibly together with $SU(2)_R$ or $U(1)_R$. The former was already worked out in [26], where it was shown that the potentials $P^{(R)}$ and $P = P^{(T)} + P^{(R)}$ do not have any critical points at all and $P^{(T)}$ only has Minkowski vacua. We now investigate the latter gauging.

We restrict ourselves to $\tilde{n} = 3$, so the group $SO(2)$ acts on A^2_μ and A^3_μ. These vectors therefore have to be dualized to tensors. We decompose the index $\tilde{I}$ as follows

$$\tilde{I} = (I, M), \tag{5.21}$$

with $I, J, K, \ldots = 0, 1$ and $M, N, P, \ldots = 2, 3$. The vector A^1_μ will act as gauge field for $SO(2)$ since its the only vector left that couples to the tensor fields. For the $U(1)_R$-gauging we take the gauge vector $A_\mu[U(1)_R] = V_I A^I_\mu$. The constraint (2.16) is automatically fulfilled.

The potential $P^{(T)}$ then becomes (taking $\Omega^{23} = -\Omega^{32} = -1$):

$$\Lambda^M_{1N} = \frac{2}{\sqrt{6}}\Omega^{MR}C_{1RN} = -\frac{1}{2}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \tag{5.22}$$

$$P^{(T)} = \frac{3\sqrt{6}}{16}h^I\Lambda^{MN}_I h_M h_N = \frac{1}{8}(\varphi^1)^5\left[(\varphi^2)^2 + (\varphi^3)^2\right]. \tag{5.23}$$

The calculation of $P^{(R)}$ however is a bit more involved since for the non-Jordan theories $C^{\tilde{I}\tilde{J}\tilde{K}}$ is not constant any more. The indices are raised using $a^{\tilde{I}\tilde{J}}$ with

$$a^{\tilde{I}\tilde{J}} = h^{\tilde{I}}h^{\tilde{J}} + h^{\tilde{I}}_{\tilde{x}}h^{\tilde{J}}_{\tilde{y}}\, g^{\tilde{x}\tilde{y}} \tag{5.24}$$

and $g^{\tilde{x}\tilde{y}} = Diag\left((\varphi^1)^2/3,\ 1/(\varphi^1)^3,\ 1/(\varphi^1)^3\right)$. Then $P^{(R)}$ becomes

$$P^{(R)} = -4C^{IJ\tilde{K}}V_I V_J h_{\tilde{K}} = -4\sqrt{2}\frac{V_0 V_1}{\varphi^1} - (\varphi^1)^2\left[V_0\left((\varphi^2)^2 + (\varphi^3)^2\right) + \sqrt{2}V_1\right]^2. \tag{5.25}$$

We remark that we have to restrict to $\varphi^1 > 0$ in order for $g_{\tilde{x}\tilde{y}}$ to be positive definite.

As already mentioned, $P^{(T)}$ only has Minkowski ground states. Moreover, $P^{(R)}$ can at most have unstable de Sitter vacua, as we proved in section 3.1. We will now study the total potential $P^{(T)} + P^{(R)}$.

The critical points of P

The total potential is

$$P = \frac{1}{8}(\varphi^1)^5 \left[(\varphi^2)^2 + (\varphi^3)^2\right] - 4\sqrt{2}\frac{V_0 V_1}{\varphi^1} - (\varphi^1)^2 A^2 \,, \tag{5.26}$$

with

$$A = V_0 \left[(\varphi^2)^2 + (\varphi^3)^2\right] + \sqrt{2} V_1 \,. \tag{5.27}$$

The first derivatives are

$$\begin{aligned} P_{,1} &= \tfrac{5}{8}(\varphi^1)^4 \left[(\varphi^2)^2 + (\varphi^3)^2\right] + 4\sqrt{2}\frac{V_0 V_1}{(\varphi^1)^2} - 2\varphi^1 A^2 \,, && (5.28) \\ P_{,2} &= \tfrac{1}{4}(\varphi^1)^2 \varphi^2 \left[(\varphi^1)^3 - 16 A V_0\right] \,, && (5.29) \\ P_{,3} &= \tfrac{1}{4}(\varphi^1)^2 \varphi^3 \left[(\varphi^1)^3 - 16 A V_0\right] \,. && (5.30) \end{aligned}$$

From (5.29) and (5.30) we get the following three possibilities for the critical points:

- When $\varphi^2 = \varphi^3 = 0$ and $V_1 = 0$, equations (5.28)-(5.30) are fulfilled and $P(\varphi_c) = 0$, giving Minkowski vacua. Since $V_I h^I_{\tilde{x}}(\varphi_c) \neq 0$, supersymmetry is broken unless also $V_0 = 0$.

- When $\varphi^2 = \varphi^3 = 0$ and $V_1 \neq 0$, equation(5.28) leads to the condition

$$(\varphi^1)^3 = \sqrt{2}\frac{V_0}{V_1} \,. \tag{5.31}$$

  Then

$$P(\varphi_c) = -6(\varphi^1)^2 V_1^2 \tag{5.32}$$

  and we have an Anti-de Sitter vacuum. The vectors $V_I h^I_{\tilde{x}}(\varphi_c)$ and $h_{Mx}\Omega^{MN} h_N(\varphi_c)$ are now identically zero, which means the vacuum preserves the full $N = 2$ supersymmetry.

- The third possibility is $(\varphi^1)^3 = 16 A V_0$, which can be rewritten as

$$V_1 = \frac{p - 16 V_0^2 q}{16\sqrt{2} V_0} \,, \tag{5.33}$$

  with $p = (\varphi^1)^3$ and $q = (\varphi^2)^2 + (\varphi^3)^2 > 0$. Using this in (5.28) and solving for V_0 gives us the following four solutions

$$V_0 = \pm\frac{1}{8}\sqrt{5p^2 + \frac{2p}{q} \pm \frac{\sqrt{4p^2 + 12p^3 q + 25 p^4 q^2}}{q}} \,. \tag{5.34}$$

Remark that the expressions under the square roots are always positive. We substitute (5.33) and (5.34) into the potential P and get

$$P(\varphi_c) = \frac{3p^{5/3}q(p + 5p^2q \pm \sqrt{4p^2 + 12p^3q + 25p^4q^2})}{4\left(2p + 5p^2q \pm \sqrt{4p^2 + 12p^3q + 25p^4q^2}\right)} . \tag{5.35}$$

Here both signs are positive when the second sign choice in (5.34) is positive, otherwise both signs are negative (independent of the choice of the first sign in (5.34)). With the plus signs we have a de Sitter vacuum, with the minus signs an anti-de Sitter vacuum.

Calculating the matrix of second derivatives P, x, y and substituting (5.33) and (5.34), we get the following form

$$P, x, y(\varphi_c) = \begin{pmatrix} B_{2\times 2} & 0 \\ 0 & 0 \end{pmatrix} . \tag{5.36}$$

The expected zero eigenvalue from the $SO(1,1)$ invariance is already explicit. Furthermore,

$$Det(B_{2\times 2}) = -\tfrac{3}{32}p^{5/3}q\left(14p + 25p^2q \pm 5\sqrt{4p^2 + 12p^3q + 25p^4q^2}\right) , \tag{5.37}$$

where again the plus sign corresponds to the de Sitter vacuum, the minus sign to the anti-de Sitter vacuum. The determinant is always negative, so there is always a negative eigenvalue and the de Sitter vacua are unstable.

5.3 Conclusions

These examples seem to suggest that the existence of stable de Sitter vacua is very model dependent. A $U(1)_R$ gauging and tensors charged under a non-compact gauge group do not guarantee stable de Sitter vacua. On the other hand, we also found a de Sitter vacuum in a model with $U(1)_R$ gauging and tensors charged under a compact group. Unfortunately the de Sitter vacuum was unstable. Whether this is a general feature of compact gaugings is not clear to us.

6 Stable de Sitter vacua with hypers

Our goal in this section is to show that it is still possible to get stable de Sitter vacua when hypermultiplets are included. We will do this by giving a particular example, namely we will gauge a specific isometry of the universal hypermultiplet. There are probably many other possibilities, but we will not analyse this in its generality here.

When there are charged hypers in the model, the potential gets some extra contributions. The total potential is given by [21, 22]

$$P = -4\vec{P} \cdot \vec{P} + 2\vec{P}^x \cdot \vec{P}_x + 2\mathcal{N}_{iA}\mathcal{N}^{iA} + P^{(T)} , \tag{6.1}$$

where, as before, $\vec{P} = h^I \vec{P}_I$, $\vec{P}_x = h^I_x \vec{P}_I$ and $\mathcal{N}^{iA} = \frac{\sqrt{6}}{4} h^I K_I^X f_X^{iA}$. Here f_X^{iA} are the quaternionic vielbeins, $f_X^{iA} f_{YiA} = g_{XY}$ with g_{XY} the metric of the quaternionic-Kähler hypermultiplet scalar manifold, K_I^X are the Killing vectors and $\vec{P}_I$ the corresponding prepotentials.

The metric of the universal hypermultiplet, together with the Killing vectors and corresponding prepotentials were given in [29], and we will repeat the results here for convenience of the reader. The four hyperscalars q^X are labelled as $\{V, \sigma, \theta, \tau\}$ and the metric is given by

$$\mathrm{d}s^2 = \frac{\mathrm{d}V^2}{2V^2} + \frac{1}{2V^2}\left(\mathrm{d}\sigma + 2\theta\,\mathrm{d}\tau - 2\tau\,\mathrm{d}\theta\right)^2 + \frac{2}{V}\left(\mathrm{d}\tau^2 + \mathrm{d}\theta^2\right) . \tag{6.2}$$

The determinant for this metric is $1/V^6$ and since the metric is positive definite in $\theta = \tau = 0$ if $V > 0$, the metric will be positive-definite and well-behaved everywhere as long as $V > 0$. This parametrization of the universal hypermultiplet is the one that comes out naturally from the Calabi-Yau compactifications of M-theory, where V acquires the meaning of the volume of the Calabi-Yau manifold. The metric (6.2) has an SU(2,1) isometry group generated by the following eight Killing vectors k_α^X

$$\begin{aligned}
&\vec{k}_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \vec{k}_2 = \begin{pmatrix} 0 \\ 2\theta \\ 0 \\ 1 \end{pmatrix}, \quad \vec{k}_3 = \begin{pmatrix} 0 \\ -2\tau \\ 1 \\ 0 \end{pmatrix}, \quad \vec{k}_4 = \begin{pmatrix} 0 \\ 0 \\ -\tau \\ \theta \end{pmatrix}, \\
&\vec{k}_5 = \begin{pmatrix} V \\ \sigma \\ \theta/2 \\ \tau/2 \end{pmatrix}, \quad \vec{k}_6 = \begin{pmatrix} 2V\sigma \\ \sigma^2 - \left(V + \theta^2 + \tau^2\right)^2 \\ \sigma\theta - \tau\left(V + \theta^2 + \tau^2\right) \\ \sigma\tau + \theta\left(V + \theta^2 + \tau^2\right) \end{pmatrix}, \\
&\vec{k}_7 = \begin{pmatrix} -2V\theta \\ -\sigma\theta + V\tau + \tau\left(\theta^2 + \tau^2\right) \\ \frac{1}{2}\left(V - \theta^2 + 3\tau^2\right) \\ -2\theta\tau - \sigma/2 \end{pmatrix}, \quad \vec{k}_8 = \begin{pmatrix} -2V\tau \\ -\sigma\tau - V\theta - \theta\left(\theta^2 + \tau^2\right) \\ -2\theta\tau + \sigma/2 \\ \frac{1}{2}\left(V + 3\theta^2 - \tau^2\right) \end{pmatrix}.
\end{aligned} \tag{6.3}$$

The corresponding prepotentials P_α^r are given by

$$\begin{aligned}
&\vec{P}_1 = \begin{pmatrix} 0 \\ 0 \\ -\frac{1}{4V} \end{pmatrix}, \quad \vec{P}_2 = \begin{pmatrix} -\frac{1}{\sqrt{V}} \\ 0 \\ -\frac{\theta}{V} \end{pmatrix}, \quad \vec{P}_3 = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{V}} \\ \frac{\tau}{V} \end{pmatrix}, \quad \vec{P}_4 = \begin{pmatrix} -\frac{\theta}{\sqrt{V}} \\ -\frac{\tau}{\sqrt{V}} \\ \frac{1}{2} - \frac{\theta^2+\tau^2}{2V} \end{pmatrix}, \\
&\vec{P}_5 = \begin{pmatrix} -\frac{\tau}{2\sqrt{V}} \\ \frac{\theta}{2\sqrt{V}} \\ -\frac{\sigma}{4V} \end{pmatrix}, \quad \vec{P}_6 = \begin{pmatrix} -\frac{1}{\sqrt{V}}\left[\sigma\tau + \theta\left(-V + \theta^2 + \tau^2\right)\right] \\ \frac{1}{\sqrt{V}}\left[\sigma\theta - \tau\left(-V + \theta^2 + \tau^2\right)\right] \\ -\frac{V}{4} - \frac{1}{4V}\left[\sigma^2 + \left(\theta^2 + \tau^2\right)^2\right] + \frac{3}{2}\left(\theta^2 + \tau^2\right) \end{pmatrix}, \\
&\vec{P}_7 = \begin{pmatrix} \frac{4\theta\tau + \sigma}{2\sqrt{V}} \\ \frac{3\tau^2 - \theta^2}{2\sqrt{V}} - \frac{\sqrt{V}}{2} \\ -\frac{3}{2}\tau + \frac{1}{2V}\left[\sigma\theta + \tau\left(\theta^2 + \tau^2\right)\right] \end{pmatrix}, \quad \vec{P}_8 = \begin{pmatrix} -\frac{3\theta^2 - \tau^2}{2\sqrt{V}} + \frac{\sqrt{V}}{2} \\ \frac{\sigma - 4\theta\tau}{2\sqrt{V}} \\ \frac{3}{2}\theta + \frac{1}{2V}\left[\sigma\tau - \theta\left(\theta^2 + \tau^2\right)\right] \end{pmatrix}.
\end{aligned} \tag{6.4}$$

The Killing vectors K_I^X are now given by $V_I^\alpha k_\alpha^X$, where the components of the embedding matrix V_I^α are constants that determine which isometries are gauged and which vector fields are used to gauge them. The corresponding prepotentials $\vec{P}_I$ then become $V_I^\alpha \vec{P}_\alpha$.

We are now ready to give a concrete example. We choose to gauge the $U(1)$ (hypermultiplet) isometry generated by $2\vec{k}_4 - \vec{k}_1 - \vec{k}_6$, so we take

$$\vec{K}_I = V_I(2\vec{k}_4 - \vec{k}_1 - \vec{k}_6)\,, \quad \vec{P}_I = V_I(2\vec{P}_4 - \vec{P}_1 - \vec{P}_6) = V_I\vec{Q}\,, \tag{6.5}$$

where we have defined $\vec{Q} \equiv 2\vec{P}_4 - \vec{P}_1 - \vec{P}_6$. For the scalar manifold of the vector multiplets we choose $\mathcal{M} = SO(\tilde{n}-1,1) \times SO(1,1)/SO(\tilde{n}-1), \tilde{n} \geq 1$, and again gauge a noncompact $SO(1,1)$ isometry of this manifold by dualizing the two charged vector fields to tensor fields (see section 4 for notation and more details). For simplicity, we do not charge the hypers under this symmetry. Our gauge group is thus $SO(1,1) \times U(1)$, where two tensors are charged under the $SO(1,1)$ and the hypers are charged under the $U(1)$. We then find that

$$\frac{\partial P}{\partial q^X}(\varphi, q_c) = 0\,, \tag{6.6}$$

$$P(\varphi, q_c) = \frac{9}{4}P^{(R)}(\varphi) + P^{(T)}(\varphi)\,, \tag{6.7}$$

where $q_c = \{V = 1, \sigma = 0, \theta = 0, \tau = 0\}$ and with $P^{(R)}$ and $P^{(T)}$ given in equations (4.8) and (4.7) respectively.

To verify this, first notice that $K_I^X|_{q_c} = 0$ and therefore

$$\mathcal{N}_{iA}|_{q_c} = 0\,, \qquad \frac{\partial(\mathcal{N}_{iA}\mathcal{N}^{iA})}{\partial q^X}|_{q_c} = 0\,. \tag{6.8}$$

We also have $\frac{\partial P^{(T)}}{\partial q^X} = 0$ since $P^{(T)}$ only depends on the scalars of the vector multiplets. The remaining term $-4\vec{P} \cdot \vec{P} + 2\vec{P}^x \cdot \vec{P}_x$ in equation (6.1) can be written as

$$-4\vec{P} \cdot \vec{P}(\varphi, q) + 2\vec{P}^x \cdot \vec{P}_x(\varphi, q) = -4C^{IJK}V_IV_Jh_K(\varphi)\vec{Q} \cdot \vec{Q}(q)\,, \tag{6.9}$$

which shows that the φ (vector multiplet) and q (hypermultiplet) dependence of this part factorizes. Since $\vec{Q}|_{q_c} = (0, 0, 3/2)$, to verify equation (6.6) we only need to check that $\frac{\partial Q^3}{\partial q^X}|_{q_c} = 0$, with Q^3 the third component of the vector $\vec{Q}$. Because Q^3 is quadratic in θ, σ and τ we have

$$\frac{\partial Q^3}{\partial \theta} = \frac{\partial Q^3}{\partial \sigma} = \frac{\partial Q^3}{\partial \tau} = 0 \quad \text{if} \quad \theta = \sigma = \tau = 0\,. \tag{6.10}$$

Finally

$$Q^3|_{\theta=\sigma=\tau=0} = 1 + 1/4V + V/4\,, \tag{6.11}$$

and it's easy to see that $V = 1$ is an extremum. This proofs equations (6.6) en (6.7).

We thus find that in the point q_c, up to a factor $9/4$ which can be absorbed in the V_I, the potential reduces to the same potential for the vector multiplet scalars as found in section 4, where we gauged a $U(1)_R$ symmetry. We now have to calculate the mass matrix in the critical point. Since (6.6) is true for any value of the vector multiplet scalars, we have

$$\frac{\partial^2 P}{\partial q^X \partial \varphi^x}|_{q_c} = 0\,, \tag{6.12}$$

so we can calculate the masses of the hyper-scalars and the vector-scalars separately. Since we already calculated the mass matrix for the vector multiplet scalars, we just have to calculate the mass matrix for the hypers. After a straightforward calculation we find the matrix to be diagonal, with all entries always positive. There are only 2 different diagonal elements and they can both be written as a sum of manifestly positive terms. Because the expressions are quite messy, we will not give them in their generality here. Instead we will look at the particular case $V_0 = 0$. We then have $\varphi_i = 0$ in the critical point and the expressions simplify significantly. Concretely, we find

$$\frac{\partial_X \partial^Y P}{P}\bigg|_c = \begin{pmatrix} 8/9 & 0 & 0 & 0 \\ 0 & 8/9 & 0 & 0 \\ 0 & 0 & 4/9 & 0 \\ 0 & 0 & 0 & 4/9 \end{pmatrix} , \tag{6.13}$$

where derivation with respect to q^X is denoted by ∂_X and indices are raised with the (inverse) quaternionic-Kähler metric g^{XY}. This shows that potentials with stable de Sitter vacua also exist when hypers are included.

7 Summary

In this paper we investigated different possibilities to get stable de Sitter vacua in $5D$ $N = 2$ gauged supergravity. We proved that $U(1)_R$ gauging (without tensors) at most leads to unstable de Sitter vacua. In the case of $SU(2)_R$ gauging we found lots of theories exhibiting de Sitter extrema, but were unable to find stable de Sitter vacua. However, by also introducing tensor multiplets and gauging a non-compact symmetry group together with the R-symmetry group we managed to construct 5D supergravity theories with stable de Sitter vacua. The used ingredients are however not sufficient to guarantee stable de Sitter vacua, as the analysis of several other examples made clear. Finally, we showed with a specific example that we can also get stable de Sitter vacua if we replace the R-symmetry gauging with charged hypers.

There are several directions in which we plan to continue our research. First of all it would be interesting to find out under which conditions stable de Sitter vacua exist in supergravity theories. A more general analysis of the potentials coming from $SU(2)_R$ gauging and tensors will certainly be useful for this. Investigating the potential coming from charged hypermultiplets might also give interesting results. Another possible fruitful path would be to try to embed the stable de Sitter vacua in $N = 4$ and $N = 8$ supergravity and check whether they are still stable. Attempts in this direction in 4D have failed (see [8]), and it would be interesting to see whether this generalizes to higher dimensions. Having an N=8 embedding could also make it easier to find their 10 or 11 dimensional origin, if any. Finally, considering the similarities between 4D and 5D $N = 2$ supergravity, the results we found perhaps suggest that investigating 4D gauged supergravities with tensor couplings might lead to new examples of stable de Sitter vacua. We hope to come back on these issues in the near future.

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A Very special real geometry

In this appendix we will repeat some elements of very special real geometry for convenience of the reader. This presentation is mostly based on the appendix in [22] and the classic paper on $5D$ $N=2$ supergravity [18].

Very special real manifolds are the scalar manifolds of $N=2$ $D=5$ supergravity coupled to vector(/tensor) multiplets and are completely determined by a symmetric 3-tensor C_{IJK}. Let M be the following $n+1$ dimensional subspace of $\mathbb{R}^{n+1}$

$$M = \{h^I \in \mathbb{R}^{n+1} | N(h) = C_{IJK} h^I h^J h^K > 0\} , \tag{A.1}$$

with metric

$$a_{IJ} = -\frac{1}{3} \partial_I \partial_J \ln N(h) . \tag{A.2}$$

Then the very special real manifold $\mathcal{M}_n$ can be defined as the hypersurface $N(h) = 1$ with metric the induced metric from the embedding space M,

$$g_{xy} = \frac{3}{2} \mathring{a}_{IJ} h^I_{,x} h^J_{,y} = -3 C_{IJK} h^I h^J_{,x} h^K_{,y} , \tag{A.3}$$

with $h^I(\phi)$ obeying $C_{IJK} h^I(\phi) h^J(\phi) h^K(\phi) = 1$, $,x$ denoting an ordinary derivative with respect to ϕ^x and [4]

$$\mathring{a}_{IJ} \equiv a_{IJ}|_{N=1} = -2 C_{IJK} h^K + 3 h_I h_J , \quad h_I \equiv C_{IJK} h^J h^K = \mathring{a}_{IJ} h^J . \tag{A.4}$$

Defining

$$h^I_x \equiv -\sqrt{\tfrac{3}{2}} h^I_{,x}(\phi) , \tag{A.5}$$

we have $h_I h^I_x = 0$, leading to

$$h_{Ix} \equiv \mathring{a}_{IJ} h^J_x = \sqrt{\tfrac{3}{2}} h_{I,x}(\phi) , \quad h^I h_{Ix} = 0 . \tag{A.6}$$

[4] It should be understood that the h^I obey $N(h) = 1$ from here on.

Using the above relations we can also write $\mathring{a}_{IJ}$ as

$$\mathring{a}_{IJ} = h_I h_J + h_I^x h_{Jx}\,, \tag{A.7}$$

and we have

$$\begin{aligned} h_{Ix;y} &= \sqrt{\tfrac{2}{3}}\left(h_I g_{xy} + T_{xyz} h_I^z\right)\,, \\ h^I{}_{x;y} &= -\sqrt{\tfrac{2}{3}}\left(h^I g_{xy} + T_{xyz} h^{Iz}\right)\,, \end{aligned} \tag{A.8}$$

where ';' is a covariant derivative using the Christoffel connection calculated from the metric g_{xy}, with

$$T_{xyz} \equiv C_{IJK} h_x^I h_y^J h_z^K\,. \tag{A.9}$$

The previous relations can be used to derive some useful identities.

Comparing (A.7) and (A.4), we obtain

$$h_I^x h_{Jx} = -2C_{IJK} h^K + 2h_I h_J\,, \tag{A.10}$$

and taking the covariant derivative with respect to ϕ^y of (A.10) leads to

$$T_{xyz} h_I^x h_J^z = C_{IJL} h_y^L + h_{(I} h_{J)y}\,. \tag{A.11}$$

Finally, after a straightforward calculation, we get

$$T_{xyz;u} = \sqrt{\tfrac{3}{2}}\left[g_{(xy} g_{z)u} - 2T_{(xy}{}^w T_{z)uw}\right]\,, \tag{A.12}$$

$$C^{IJK}{}_{,x} = 2T_{uvw;x} h^{Iu} h^{Jv} h^{Kw}\,. \tag{A.13}$$

This formula was found in [24], but with an erroneous factor 3 instead of 2.

The domain of the variables should be limited to $h^I(\phi) \neq 0$ and the metrics $\mathring{a}_{IJ}$ and g_{xy} should be positive definite. The latter two conditions are equivalent.

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